arbitrary profiles for f and H, each of which involves an arbitrary function of ξ, and to require that the integrated Eqs. (19) and (20) be satisfied along with Eqs. (9, 21–24, and 27). The latter alternative was adopted in Ref. 2, and machine computations are to be made using that approach. Approximate profiles for f and H may be computed from Eqs. (17) and (18), employing the parameters obtained from the foregoing computations.

4. Closure

It is recognized that the substantiation of the results obtained by an analysis such as that discussed in the foregoing must be based on comparisons with experimental data. A complete discussion of the nature of the approximation is beyond the scope of the note presented herein. Additional information may be found in Ref. 2.

References

[1] Hayes, W. D., "On laminar boundary layers with heat transfer," Jet Propulsion **26**, 270–274 (1956).

[2] Skifstad, J. G., "Approximate analysis of an unconfined electric arc in forced convection with application to tube-arc heaters," TM-62-11, Jet Propulsion Center, Purdue Univ. (December 1962).

[3] Pai, S., *Viscous Flow Theory: Laminar Flow* (D. Van Nostrand Co., Inc., New York, 1956), Vol. I, Chap. X.

Temperature Measurement of Hot Gas Streams

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An experimental method is presented with which the temperature of equilibrium, nonradiating, hot gas streams can be measured. The method, consisting of a small probe and companion equations, has been used to measure the temperature of the combustion products of the cyanogen-oxygen flame ($T \approx 7800°R$) at 1 atm pressure; results are within 3% of spectrographic and microwave temperature determinations.

Nomenclature

A	= area, ft²
C	= specific heat of wall material, Btu/lb-°R
C_p	= specific heat of gas adjacent to probe wall at constant pressure, Btu/lb-°R
D	= diameter of probe, ft
h_e,h_w	= enthalpy at freestream, probe wall, Btu/lb
k	= thermal conductivity of probe wall, Btu/fps-°R
l	= thickness of probe wall, ft
M	= molecular weight
N_{Pr}	= Prandtl number of gas adjacent to wall, $(C_p\mu/k)_w$
$P_t\,P$	= total, static pressure of freestream gas, psia
q	= heat transfer coefficient, Btu/sec-ft²-°R
r	= nose radius, ft
R	= universal gas constant
T_w,T_0	= temperature of probe wall, initial temperature, °R
u	= velocity of freestream, fps
du/dx	= inviscid velocity gradient at stagnation point, sec⁻¹
x	= distance along probe face, ft
Z	= factor defined by Eq. (4)
γ	= ratio of specific heats $C_p/(C_p - R)$
μ	= viscosity at freestream, lb/ft-sec
ρ,ρ_w	= density of gas at freestream, density of probe wall material, lb/ft³
τ	= time, sec

Received January 28, 1963.
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The transient heat transfer technique long has been used by shock-tube and wind-tunnel experimenters to determine the rate of heat transfer from a thermodynamically specifiable test medium to a specific test model. Essentially, the transient technique is a calorimetric method where the heat transfer rate is determined by measuring the time rate of change of the temperature at a point on the model. This technique can be used equally well to measure the rate of heat transfer to a probe from a gas stream whose temperature is to be determined. The main advantage of using the transient technique is that it allows short testing time (of the order of seconds) which, when testing in high-temperature gas streams, eliminates the need for probe cooling.

The temperature of the gas stream is determined uniquely from the heat transfer coefficient in the solution of the Prandtl boundary layer equations. This heat transfer coefficient is defined as the heat transfer rate divided by the difference between the enthalpy of the freestream and that of the gas adjacent to the probe surface. The justification lies in the fact that for simple shapes (axisymmetric stagnation points, flat plate, etc.) the boundary layer equations yield exact solutions within the general boundary layer approximation, even for compressible flow with variable transport properties. Thus, by determining the heat transfer rate experimentally and knowing the enthalpy of the gas adjacent to the probe wall, it is possible to solve directly for the freestream enthalpy and, therefore, the freestream temperature.

The rate at which heat is transferred to a probe, assuming that the probe wall has an infinite thermal conductivity, is

$$dQ/d\tau = \rho_w AlC[\partial(\Delta T)/\partial\tau] \tag{1}$$

At the stagnation point of the probe, Newton's heat transfer equation may be written as

$$dQ/d\tau = qA[h_e + (u^2/2) - h_w] \tag{2}$$

To take into account the actual temperature difference existing across the front face of the probe due to the finite thermal conductivity of the probe wall Z, a correction factor on Eq. (1) is introduced (following Ref. 1):

$$(dQ/d\tau)_{\text{actual}} = Z(dQ/d\tau) \tag{3}$$

For a perfectly insulated wall,

$$Z \approx [1 - (lq/3k)]^{-1} \tag{4}$$

Combining Eqs. (1–3) gives

$$h_e = [Z\rho_w lc/q](\partial T_w/\partial\tau) + h_w - (u^2/2) \tag{5}$$

The stagnation point heat transfer coefficient q, which takes into account real-gas effects for equilibrium air, can be obtained from Ref. 2. However, for an incompressible perfect gas, the heat transfer coefficient has been determined to be[3]

$$q = 0.763(N_{Pr})^{-0.6}(\rho\mu)^{0.5}(du/dx)^{0.5} \tag{6}$$

Since comparison of Eq. (6) with Eq. (79) of Ref. 2 will show that both real-gas effects and compressibility effects have only a small influence when evaluated for velocities below about 21,000 fps, the use of Eq. (6) generally will suffice.

The value of the velocity gradient for a probe having a hemispherical nose in supersonic flow, found from a modified Newtonian flow theory consideration, is

$$du/dx = (1/r)[2(P_t - P)/P_t]^{0.5} \tag{7}$$

The velocity gradient for low subsonic velocities is obtained from potential-flow theory. For a hemispherical nosed probe in incompressible flow,

$$du/dx = 3u/D \tag{8}$$

and, for a flat-faced probe,

$$du/dx = 4u/D \tag{9}$$

The freestream velocity u is determined as a function of gas temperature by measuring the total (static plus dynamic) and static pressure of the stream. At low values of Mach number, Bernoulli's equation can be used and gives

$$u = \{2(R/M)T[(P_t/P) - 1]\}^{0.5} \tag{10}$$

For supersonic streams, u again can be determined, as a function of gas temperature, from a knowledge of the free-stream static and total pressure by using the compressible flow relation for constant specific heat:

$$u = \left\{\frac{2\gamma}{\gamma - 1}\left(\frac{R}{M}\right) T\left[\left(\frac{P_t}{P}\right)^{(\gamma-1)/\gamma} - 1\right]\right\}^{0.5} \tag{11}$$

The values for the transport properties of each constituent of the jet can be obtained from Ref. 4 and combined to calculate the mixture properties by using the nomographs of Ref. 5. The specific heat for each component also can be obtained from Ref. 4. This procedure was used herein to measure the temperature of the combustion products of the cyanogen-oxygen flame. This is a low subsonic jet ($M < 0.2$) having an adiabatic flame temperature of about 8700°R but found from spectrographic measurements to be actually about 8000°R. (For a description of the burner and associated apparatus, see Ref. 6.)

Since the composition of the combustion products of the cyanogen-oxygen reaction, carbon monoxide and nitrogen, remains relatively constant with change in temperature,[6] that is, there is little dissociation, and since the radiative emissivities of these gases are very low in the temperature range under consideration, the primary heat transfer mode is one of convection. Therefore, only the heat transfer coefficient calculated from the boundary layer equations is required to describe fully the overall mechanism of heat transfer. A schematic diagram of the probe used for the temperature determination is shown in Fig. 1. In order to reduce heat losses, 1) the thickness of the front face l is very small compared to the diameter, and 2) the probe is evacuated. A flat-faced probe was chosen, as this configuration is relatively easy to fabricate while allowing the thickness l to be machined to a very high degree of uniformity. A thermocouple welded to the stagnation point of the probe measures the wall temperature, and, by recording this with an oscillograph, the time rate of change of this temperature ($\partial T_W/\partial\tau$) is determined. The temperature of the combustion products determined by this method was found to be about 7800°R, which was within 3% of microwave attenuation and spectrographic determinations. Thus, this technique has proved to be an accurate and relatively simple means to measure the temperature of high-temperature gas streams.

References

[1] Trimpi, R. L. and Jones, R. A., "Transient temperature distribution in a two-component semi-infinite composite slab of arbitrary materials subjected to aerodynamic heating with a discontinuous change in equilibrium temperature or heat-transfer coefficient," NACA TN 4308 (September 1958).

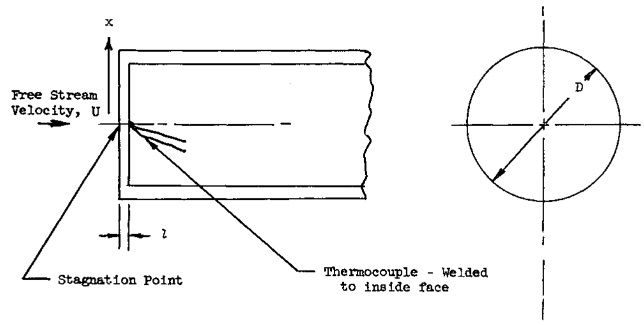

Fig. 1. Calorimeter probe.

[2] Cohen, N. B., "Boundary-layer similar solution and correlation equations for laminar heat-transfer distribution in equilibrium air at velocities up to 41,100 feet per second," NASA TR-118 (1961).

[3] Sibulkin, M., "Heat transfer near the forward stagnation point of a body of revolution," J. Aeronaut. Sci. **19**, 570–571 (1952).

[4] Svehla, R. A., "Estimated viscosities and thermal conductivities of gases at high temperatures," NASA Rept. R-132 (1962).

[5] Brokaw, R. S., "Alignment charts for transport properties, viscosity, thermal conductivity and diffusion coefficient for non-polar bases and gas mixtures at low density," NASA Rept. TR R-81 (1961).

[6] Huber, P. W. and Gooderum, P. B., "Experiment with plasmas produced by potassium-seeded cyanogen oxygen flames for study of radio transmission at simulated reentry vehicle plasma conditions," NASA TN D-627 (January 1961).

On the Stability of a Class of Discontinuous Attitude Control Systems

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An examination is made of the stability, under external torque, of attitude control systems having fixed minimum-impulse bits delivered at unsymmetrical vertical switch lines. A difference equation is obtained for successive intersections of a single switch line. The Liapunov method is used to show that any steady external torque causes instability.

Nomenclature

c = $L_c/(L_c + L_e)$
e = attitude error
$\Delta\dot{e}$ = change in error rate due to a fixed control impulse bit
I = moment of inertia
L_c = control torque
L_e = external torque
x = dimensionless perturbation attitude error rate

A PARTICULAR class of discontinuous attitude control systems has been examined for space vehicle applications by several authors.[1, 2] These systems are characterized by fixed-impulse bits delivered at unsymmetrical vertical switch lines in the error, error-rate phase plane. Figure 1 shows the limit-cycle behavior achieved by such systems under special conditions (no external torque, plant having inertia only), after a convergence period that differs depending upon the system details. It is the purpose of this note to point out that any steady external torque causes these systems to become unstable. That is, the trajectory eventually "escapes" from the inner switch lines.

Two types of trajectory may exist under external torque:

1) The control-torque-off trajectory following a fixed control impulse either intersects the same switch line or neither switch line. In the latter event, the trajectory "escapes," and a limit cycle is possible only if additional symmetrical or unsymmetrical switch lines are provided at larger (absolute) values of error. But then the behavior is unstable according to the criterion adopted and need not be considered further.

2) The control-torque-off trajectory following a fixed control impulse intersects the opposite switch line.

Received April 15, 1963. This work was conducted under the Douglas Aircraft Company Inc. Independent Research and Development Program Account No. 81335-172.

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